APPLIED STATISTICS

ESTIMATING THE PROBABILITIES OF A SHARK ATTACK WHEN USING AN ELECTRIC REPELLENT

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Key words: Bayesian estimation; *Carcharodon carcharias*; electrical repulsion; great white; jackknife; number of approaches; point and interval estimation; shark, SharkPOD; standard error of estimate; time to attack; Type I censoring.

Summary: In two series of tests of a new electric shark repellent (the SharkPOD), data was collected on the time needed to attack the bait, under power-off and power-on (active) conditions. Various approaches were followed to estimate the relevant probabilities and their standard errors, e.g. through Bayesian updating and resampling methods (especially jackknifing). Conclusions were separately drawn after completion of the first experiment (in which there were 8 successful attacks in 98 five minute active periods), and after completion of the second experiment (in which no successful attacks were recorded in 24 ten minute active periods). In general it was concluded that the probability of an attack in at most 5 minutes was reduced from about 0.70 in power-off mode to about 0.08 in power-on mode and in a period of at most 10 minutes from 0.90 to 0.16.

1. Introduction

Worldwide there are probably 70 to 100 shark attacks annually resulting in about 5–15 deaths (International Shark Attack File, 1997). Even though
shark attacks are a minor cause of mortality for humans, this phenomenon receives an inordinate amount of media cover and interest, probably due to humans’ psychological abhorrence of being eaten alive. Shark protection programs have been attempted in various forms, including the installation of anti-shark nets at bathing beaches, bubble curtains, chemical repellents and shark screens (Johnson, 1987). Protection of divers has also received much attention, particularly as shark diving has become a substantial tourist industry over the past few years (Tzimoulis, 1997). This protection has taken the form of using explosive devices, “powerheads”, clubs or “shark billies” and protective “chain-mail” suits (Johnson, 1987).

In South Africa, shark attack on humans has been perceived to be a problem since the early 1900s with a minimum of 103 shark interactions with humans resulting in personal injury over the past 30 years (Cliff, 1991). Anti-shark nets were first installed off KwaZulu-Natal in 1952 and have proved to be extremely effective with an apparent 91% reduction in shark attack frequency from 1.08 to 0.10 serious attacks per year (Dudley, 1997). However, there is concern about possible environmental impacts of the shark nets on the nearshore ecosystem (Dudley and Cliff, 1993). The CSIR initiated South African testing of electrical fields to protect bathers, but this program was discontinued in 1988 (Cliff, 1988). Subsequently, the Sharks Board has investigated the concept of using electrical fields to repel sharks, employing a unique electrical configuration in an attempt to create an efficient, yet small protective device (SharkPOD). The following field trials were conducted to test the SharkPOD against the shark species most regularly implicated in injurious interactions with humans, the great white Carcharodon carcharias (Cliff, 1991).
2. Description of experiments

The experiments were conducted in the channel between Dyer Island and Geyser Rock, Western Cape, South Africa, an area renowned for the presence of white sharks patrolling the area preying on Cape fur seals (*Arctocephalus pusillus*) that have ventured away from the colony on Geyser Rock.

Experimental design:

The aim of the experiment was to investigate whether a great white shark enticed to feeding could be kept away from a static bait through the SharkPOD. Two experiments were conducted: The first during March 1995 and the second from August to September 1997. In Experiment 1 Cape fur seal carcasses were used as bait, but due to restrictions imposed by national government institutions, soupfin shark *Galeorhinus galeus* was used for the second experiment.

For the experimental setting, the bait was attached to the SharkPOD, consisting of two electrodes which could be energized by a switch. The electrodes were attached to the bottom of a floating rig that kept the electrodes 1.5 m apart with the bait positioned between them. This rig was connected by a 10 m rope to the skiboat on which the observers were stationed. For the control setting, the rig and bait had the same outward appearance as for the experimental case. The rigs could easily be deployed and hauled in, using the rope.

Individual sharks were identified through visible natural features, e.g. scarring. A shark usually approaches and swims past the bait several times before attempting to bite and feed on it. An approach leading to an attack on the bait is called a “successful approach”. We use the term “approaches” to include passes and the actual feeding events.
A cross-over design was applied as follows: The control bait was first set out and remained in the water to attract a shark, called the running-in period. The appearance of a shark then initiated the so-called control period. When the shark was clearly interested in feeding on the bait, the bait was hauled away from the biting shark toward the boat and the experimental bait released in its place, starting the experimental period. The shark was exposed to this energized POD bait for only short periods (five minutes during Experiment 1 – March 1995 and ten minutes during Experiment 2 – August/September, 1997), with the intention to ensure that the shark did not lose interest in the bait and move away from the boat. At the end of the experimental period, the control bait was again deployed to introduce the next trial, starting either with a running-in period (when no sharks were visible) or a control period (when the shark was still interested in feeding). The procedure was repeated for as many trials as possible, until the sharks stayed away or the experiment had to be terminated for other reasons (bad weather or poor visibility, etc.). All observations were made from a 2.5 m high platform attached to the steering console of the skiboot. Real-time records were kept of the start of each trial and the time at which each pass or attack event occurred, as well as detailed descriptions of the event.

3. Analysis of experiment 1 (Prototype SharkPOD™)

3.1 Data description and assumptions

The information collected during Experiment 1 consisted of information records, each containing data on the following variables:

- Serial number, ranging from 1 to 98.
- The date of data collection, ranging from 2 March 1995 to 31 March 1995.
The specific metal of which the electrodes (between which the bait was attached) were made (called the electrode configuration), namely stainless steel plate, titanium and stainless steel mesh.

- Status of the experiment, namely running-in, power-off (control) and power-on.
- Trial number, identical to the serial number.
- Time of the day when measurements were recorded.
- Time needed until bait was taken during the specific trial period, ranging from 0.5 minutes to 19 minutes.
- Number of approaches (attacks, attempts and passes), ranging from 0 to 20.
- Closest distance that a shark has come to the bait, ranging from 0 m to 6 m, with two exceptions of 15 m and 20 m.
- Feeding taking place, “Yes” or “No”.
- Comments, e.g. when running-in, control or experimental phases started, how sharks behaved, whether a failure occurred (feeding in on-mode), etc.

The data format did not lend itself to basic analyses and therefore the most important information was carried over to an Excel spreadsheet, containing records on the following variables: Date, trial number, power status (off or on only), time period (5 minutes length for experimental phase and varying length for control phase), time needed until bait was taken, the number of approaches (including actual feeding event) and closest distance to bait. Added to this information, the electrode metal-type was noted:

- trials 1 to 51 : stainless steel plate
- trials 52 to 84 : titanium
- trials 85 to 98 : stainless steel mesh

Information on water temperature and specific gravity (SG) was also collected. It was decided not to use the information on electrode structure, water temperature and SG in the analysis. This decision was based mainly on the fact that the experimental design and amount of available data did
not support the detailed level of analysis that would be required for such an investigation.

From the Excel data sheet it was clear that there were various shortcomings in the data recording, namely:

- The number of approaches in off-mode was seldom noted.
- The on-periods were of length 5 minutes only, resulting in a data series which might have been too heavily censored (Type I censoring).
- During the last 6 power-on periods in which the sharks successfully fed (attacked), the actual time needed (in the 5 minute period) was not noted. These times were assumed to be equal to 2.5 minutes each (middle of the 5 minute period). The remaining time after successful feeding in all the power-on time periods was taken to be zero, due to inaccuracy/uncertainty of measurement.
- There was a strong suspicion for some of the 8 reported cases in which feeding took place in power-on mode, that the power source had failed and that the power was actually off. It was, however, decided to assume these 8 observations represented failures of the POD.

The time needed to take the bait (feeding) and the number of approaches to take the bait (including the final approach in which the bait was taken) were the primary characteristics to be analysed in the first experiment. The power-off data therefore consisted of 98 observations of time to feed (when enticed to feeding after a running-in phase), while the power-on data consisted of 98 observations of time, and 98 observations of number of approaches, to attempt feeding. Both the latter data sets were censored to a five minute time interval.

Table 1 (excluding the last two columns) contains the sample sums, averages (arithmetic means) and standard deviations of

- time needed to take the bait;
- number of approaches to take the bait (including the final approach);
- closest distance to bait.
In power-on mode, the time needed for attack was assumed to have an exponential distribution and the number of approaches needed for attack was assumed to have a geometric distribution. The estimates of the mean and standard deviation in the table for these two variables were based on Type I censoring from the respective distributions.

Table 1. Summary statistics for power-off and power-on modes, Experiment 1 (n = 98 trials)

<table>
<thead>
<tr>
<th>Power mode</th>
<th>Time taken to feed (minutes)</th>
<th>Number of approaches to feed</th>
<th>Closest distance to bait (metre)</th>
<th>Time to failure (TTF) (minutes)</th>
<th>Number of approaches to feed (ATF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Off</td>
<td>Sum 339.5</td>
<td>$\downarrow_1$</td>
<td>0$\downarrow_2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average 3.46</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Std dev 2.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On</td>
<td>Sum 468.0$\downarrow_3$</td>
<td>500$\downarrow_4$</td>
<td>298.5</td>
<td>463$\downarrow_5$</td>
<td>528$\downarrow_5$</td>
</tr>
<tr>
<td></td>
<td>Average 58.5$\downarrow_3$</td>
<td>63.50$\downarrow_4$</td>
<td>3.05</td>
<td>57.88$\downarrow_5$</td>
<td>66.00$\downarrow_5$</td>
</tr>
<tr>
<td></td>
<td>Std dev 58.5$\downarrow_3$</td>
<td>57.73$\downarrow_4$</td>
<td>3.09</td>
<td>48.98$\downarrow_5$</td>
<td>68.79$\downarrow_5$</td>
</tr>
</tbody>
</table>

$\downarrow_1$ Not reported on data sheet
$\downarrow_2$ All values equal to zero
$\downarrow_3$ Based on exponential model with Type I censoring
$\downarrow_4$ Based on geometric model with Type I censoring
$\downarrow_5$ Based on Assumption A (see below)

The summary statistics for closest distance to bait clearly illustrate the effect of the electrical field generated by the device. This distance variable will not be analysed further for Experiment 1.

Since it was felt that the Type I censoring in power-on mode limited information too heavily on time and number of approaches to feeding, it was decided to also investigate the power-on data under a different approach. This analysis was accomplished through Assumption A:
Assumption A: The sequence of 5 minute intervals in power-on mode can be connected in a single continuous time interval of \( 98 \times 5 = 490 \) minutes in which the shark attacks were recorded as random events.

The rationale behind Assumption A is as follows: The experiment was conducted in such a manner that a shark was first attracted (running-in) and then enticed to feeding in power-off mode of sufficient time length. Then the control rig was quickly towed in, the power switched on and the experimental rig deployed for 5 minutes. It was argued that the consecutive 5 minute on-periods could then be seen as a continuous time period without censoring.

Using Assumption A, we could now derive two additional variables, namely time-to-failure/feeding (TTF) and approaches-to-failure/feeding (ATF) (both for power-on mode only and up to the eighth failure). The 8 observations for these two variables were

- TTF: 41, 37, 52.5, 97.5, 162.5, 32.5, 22.5, 17.5 (total = 463 minutes)
- ATF: 46, 10, 33, 136, 207, 44, 30, 22 (total = 528 approaches)

with respective averages and standard deviations given in the last two columns of Table 1.

Comparison of the averages and standard deviations for time to attack and number of approaches to attack (both in power-on mode), shows a fairly reasonable agreement between the results for Type I censoring and those under Assumption A, giving some justification for the results of the Type I censoring.

3.2 Statistical modelling and applications

Our purpose is now to estimate certain probabilities of a shark attack when using the prototype SharkPOD™ and to compare these probabilities with the probabilities of an attack without using the POD. First, we will introduce some notation.
Let \( T = \) time to failure/successful feeding

\( N = \) number of approaches to failure/successful feeding

and define

\[
\pi_T(\text{off}, t) = P(T \leq t | \text{power-off})
\]

\[
\pi_T(\text{on}, t) = P(T \leq t | \text{power-on})
\]

\[
\pi_N(\text{off}, n) = P(N \leq n | \text{power-off})
\]

\[
\pi_N(\text{on}, n) = P(N \leq n | \text{power-on})
\]

Also, let \( p_T(\text{off}, t) \), \( p_T(\text{on}, t) \), \( p_N(\text{off}, n) \) and \( p_N(\text{on}, n) \) respectively denote the estimated probabilities.

**Probability estimates**

The experimental design suggests estimating the probability of a successful feeding event in at most 5 minutes. Specifically, it was seen from the data that 96 of the 98 successful feeding events in power-off mode took place in less than 5 minutes and also (as result of the Type I censoring to 5 minutes) there were 8 successful feeding events in power-on mode. Therefore, we can naively estimate these probabilities as follows:

\[
p_T(\text{off}, 5) = 96/98 = 0.980 \text{ with standard error } = 0.01428;
\]

\[
p_T(\text{on}, 5) = 8/98 = 0.082 \text{ with standard error } = 0.02768.
\]

Investigating the probability of being attacked by a shark when diving in seas where the great white shark roams, implies modelling a survival phenomenon where failures (attacks) occur randomly. The exponential time-to-failure model and the geometric approaches-to-failure model are the most obvious stochastic models for our problem. We use these models for our next estimates.
Power-off mode

For the power-off mode we have an average time-to-failure of 3.46 (see Table 1). The exponential model with mean estimated by 3.46 gives a cumulative distribution function for $T$ of $F_T(t) = 1 - e^{-\frac{t}{3.46}} = p_T(\text{off}, t)$.

The probability of a shark attack (when not using the POD, in an environment where sharks are enticed to feeding) for periods of 5 and 10 minutes, are then respectively given by $p_T(\text{off}, 5) = 0.764$ and $p_T(\text{off}, 10) = 0.944$. 

Power-on mode

For the power-on mode we have two estimates of the mean of the assumed exponential model of time-to-failure, namely $\bar{x}_T = 58.50$ (Type I censored data) and $\bar{x}_T = 57.88$ (using Assumption A). Using the same reasoning as for the power-off case, except that we now assume the POD to be activated, gives the estimated probabilities for an attack in 5 and 10 minutes respectively as $p_T(\text{on}, 5) = 0.082$, $p_T(\text{on}, 10) = 0.157$ (Type I censoring) and $p_T(\text{on}, 5) = 0.083$, $p_T(\text{on}, 10) = 0.159$ (Assumption A).

Next, we model the distribution of $N$, the number of approaches needed to attack in power-on mode. Using the maximum likelihood estimator for the parameter $\pi_N(\text{on}, 1)$ of a geometric model under Type I censoring, we find $p_N(\text{on}, 1) = \frac{8}{500+8} = 0.0157$. and the estimated mean number of approaches to failure $\bar{n} = \frac{1}{p_N(\text{on}, 1)} = 63.5$. When we apply Assumption A on the total number of approaches, given in Table 1, we get the estimate $p_N(\text{on}, 1) = \frac{8}{528} = 0.0152$ and the estimated mean number of approaches to failure $\bar{n} = \frac{1}{p_N(\text{on}, 1)} = 66$.

The geometric model can now be used to estimate the probability of a failure in at most $n$ approaches, namely $p_N(\text{on}, n) = F_N(n) = 1 - (1 - p_N(\text{on}, 1))^n$, with $F_N(n)$ the cumulative distribution function.
of $N$. Applying this formula for Type 1 censoring, for 5 and 10 approaches respectively, gives probability estimates $p_N \text{ on, 5} = 0.076$ and $p_N \text{ on, 10} = 0.146$. When using Assumption A, we obtain $p_N \text{ on, 5} = 0.074$ and $p_N \text{ on, 10} = 0.142$ respectively.

**Precision of probability estimates**

In order to evaluate the precision of the probability estimates given above, we calculated the jackknife estimates of standard errors of the probabilities. These estimates, together with the relevant probability estimates, are given in Table 2. Bootstrap estimates of the standard errors of the probabilities are very similar to the jackknife estimates given in Table 2, and are not reported here.

**Table 2.** Estimated probabilities of successful feeding and jackknife estimates of standard errors (exponential model only).

<table>
<thead>
<tr>
<th>Time/Approaches</th>
<th>Power-off</th>
<th>Power-on $p_N \text{ on, } t$</th>
<th>$p_T \text{ on, } t$</th>
<th>Standard error</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time, $t$</td>
<td>$p_T \text{ off, } t$</td>
<td>$p_T \text{ off, } t$</td>
<td>$p_N \text{ off, } n$</td>
<td>$p_N \text{ on, } n$</td>
<td>$p_N \text{ on, } n$</td>
</tr>
<tr>
<td>5</td>
<td>0.764</td>
<td>0.02211</td>
<td>0.082</td>
<td>0.00114</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.944</td>
<td>0.01027</td>
<td>0.157</td>
<td>0.00211</td>
<td></td>
</tr>
<tr>
<td>Approaches, $n$</td>
<td>$p_N \text{ off, } n$</td>
<td>$p_N \text{ off, } n$</td>
<td>$p_N \text{ on, } n$</td>
<td>$p_N \text{ on, } n$</td>
<td>$p_N \text{ on, } n$</td>
</tr>
<tr>
<td>5</td>
<td>$p_N \text{ on, } n$</td>
<td>$p_N \text{ on, } n$</td>
<td>0.076</td>
<td>0.00758</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>$p_N \text{ on, } n$</td>
<td>$p_N \text{ on, } n$</td>
<td>0.146</td>
<td>0.01399</td>
<td></td>
</tr>
</tbody>
</table>

$\text{\textbar}_1$ Type I censored data only
$\text{\textbar}_2$ Data not recorded

### 3.3 Comparison of results of probability estimation methods

Comparing the naive and exponentially modelled point estimates of the probability of a shark attack in at most 5 minutes for the power-off mode, shows a fairly great difference (0.980 vs 0.764). The implication of this difference is that we cannot put too much confidence in the estimated probabilities in power-off mode, but may guess that this probability is probably in the order of 0.76.
For an attack in at most 10 minutes, the probability could be more than 0.90 (in power-off mode).

When we compare the two approaches in modelling, namely Type I censoring and Assumption A, the estimates of the probabilities in power-on mode are found to be almost the same (0.082 vs 0.083 and 0.157 vs 0.159 for 5 and 10 minute periods respectively).

4. Analysis of Experiment 2 (SharkPOD™ Production Model)

4.1 Data description and assumptions
As it was felt that the five minute power-on periods in Experiment 1 were too short, resulting in too heavy censoring, these periods were increased to ten minutes in Experiment 2. Furthermore, each approach was now separately recorded in terms of time of the day, shark identification and closest distance from the bait. Altogether there were 24 trials, each consisting of a power-off period (some preceded by a running-in period when necessary) with various numbers of approaches, and a power-on period with various numbers of approaches. Two power-on periods had zero approaches, apparently solely due to the electrical field of the POD, as the sharks returned when the power was switched off.

In power-off mode, the time taken and the number of approaches to feed were now derived as the time/number of approaches needed to come to a zero distance from the bait for the first time during the specific trial (except for trial 1, where the closest distance was 0.5 m). In the power-on mode, the sharks never succeeded to feed on the bait, so that the censored time to feeding was 10 minutes in all 24 trials.
PROBABILITY OF A SHARK ATTACK

Table 3 gives a summary (in terms of sums, averages and standard deviations) of the time and the number of approaches needed to feed for the first time in power-off mode, while for power-on mode only the number of approaches during a ten minute period is summarised. For both modes, the closest distance from the bait is also summarised.

Table 3. Summary statistics for power-off and power-on modes, Experiment 2 \( (n = 24 \text{ trials}) \)

<table>
<thead>
<tr>
<th>Power mode</th>
<th>Time taken to feed (minutes)</th>
<th>Number of approaches ( l_1 )</th>
<th>Closest distance to bait (metre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Off:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum</td>
<td>102</td>
<td>78</td>
<td>0.5</td>
</tr>
<tr>
<td>Mean</td>
<td>4.25</td>
<td>3.25</td>
<td>0.02</td>
</tr>
<tr>
<td>Std dev</td>
<td>3.12</td>
<td>2.97</td>
<td>0.10</td>
</tr>
<tr>
<td>On:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td>191</td>
<td>57</td>
</tr>
<tr>
<td>Mean</td>
<td>( l_2 )</td>
<td>7.96</td>
<td>2.38</td>
</tr>
<tr>
<td>Std dev</td>
<td></td>
<td>6.15</td>
<td>1.27</td>
</tr>
</tbody>
</table>

\( l_1 \) Approaches to feed in *power-off* mode; all approaches in 10 minute *power-on* periods

\( l_2 \) No feeding during power-on periods for all 24 trials

As can be seen from Table 3, there can be no doubt about the effectiveness of the SharkPOD\(^{TM}\).

It now remains to find updated estimates of the probabilities of an attack in the two power modes.

4.2 Statistical modelling and applications

*Power-off mode*

We can now apply the same naive and exponentially modelled approaches used for Experiment 1 to estimate feeding probabilities in terms of time required and number of approaches required.
The probabilities and their standard errors for an attack in time periods of at most 5 and at most 10 minutes are respectively estimated naively from the relative frequencies in the data as:

\[ p_T(\text{off}, 5) = \frac{17}{24} = 0.708 \text{ with standard error } 0.09278 \]
\[ p_T(\text{off}, 10) = \frac{23}{24} = 0.958 \text{ with standard error } 0.04079 \]

Assuming an exponential model for time-to-failure and using the average time of 4.25 (from Table 3), the estimated probabilities and their jackknife standard errors are:

\[ p_T(\text{off}, 5) = 1 - e^{-5/4.25} = 0.692, \text{ standard error } 0.05465 \]
\[ p_T(\text{off}, 10) = 0.905, \text{ standard error } 0.03338 \]

Similarly, the probabilities and their standard errors for feeding in at most 5 and 10 approaches respectively are estimated naively as

\[ p_N(\text{off}, 5) = \frac{20}{24} = 0.833 \text{ with standard error } 0.07607 \]
\[ p_N(\text{off}, 10) = \frac{23}{24} = 0.958 \text{ with standard error } 0.04079 \]

Assuming a geometric distribution for the number of approaches required to first feed and using the average required number of approaches given in Table 3 as 3.25, the estimated probabilities and their jackknife standard errors for 5 and 10 approaches respectively are given by

\[ p_N(\text{off}, 5) = 1 - \left(1 - \frac{1}{3.25}\right)^5 = 0.841, \text{ standard error } 0.06609 \]
\[ p_N(\text{off}, 10) = 0.975, \text{ standard error } 0.01979 \]

Power-on mode

Since we have zero successful feeding events in all 24 power-on trials, we are now challenged to find new estimates of the probabilities that were estimated for Experiment 1. A commonly used formula for a 100(1 - \alpha)\% confidence interval for \( \pi \), the probability of a success in \( n \)
independent Bernoulli trials in which no successes were observed, is given
by \( \binom{n}{0} \left( 1 - \frac{1}{n} \right). \) Since the trials in our experiment could involve the same or different sharks, the assumption of independence of the Bernoulli trials might be suspect. However, it was felt that even for the same shark, different approaches spread over time, could be regarded as independent. We may therefore apply this formula to find a 99% confidence interval for \( \pi_T(on, 10) \), namely (0; 0.175).

By combining the results of both Experiment 1 and Experiment 2, we can also endeavour to use a Bayesian approach to update the estimated probabilities obtained after Experiment 1.

For this purpose, we argued as follows: Assume a beta prior for \( \pi_T(on, 10) \), with parameters \( \alpha \) and \( \beta \) estimated through the method of moments from Experiment 1 (see Table 2):

\[
p_T(on, 10) = 0.157 = \frac{\alpha}{\alpha + \beta} \quad \text{and} \quad (\text{standard error})^2 = (0.00211)^2 = \frac{\alpha \beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)}
\]

From these equations we find the estimates \( \hat{\alpha} = 4667.1 \) and \( \hat{\beta} = 25059.7 \). Taking the likelihood function of a Bernoulli process with 0 successes in \( n = 24 \) trials together with the prior mentioned above, standard arguments lead to a beta posterior distribution for \( \pi_T(on, 10) \) with parameters \( a = \alpha + \sum x_i = 4667.1, \quad b = \beta + n - \sum x_i = 25059.7 + 24 = 25083.7, \) where \( \sum x_i = \text{number of successes}. \)

From the posterior distribution we find the Bayes point estimate for \( \pi_T(on, 10) \) as \( p_T(on, 10) \) (Bayes) = \( \frac{\alpha}{\alpha + b} = 0.157 \) and the 99% Bayes lower confidence interval as (0; 0.162).

In an effort to complete the picture we assumed the first 5 minutes of each of the 24 ten minute periods as independent observations of \( T = \text{time to failure} \) censored to 5 minutes. We then have zero successes in 24 Bernoulli trials
with success probability $\pi_T(\text{on}, 5)$. We may then follow the same line of reasoning as above for finding a beta posterior distribution of $\pi_T(\text{on}, 5)$ with parameters $a$ and $b$, $a = 4860.8$ and $b = 53727.1$. From this distribution, the point estimate of $\pi_T(\text{on}, 5)$ is $p_T(\text{on}, 5) = 0.083$ and the 99% lower Bayes confidence interval is $(0; 0.086)$.

Finally, we used two approaches to estimate the probabilities of a successful feeding event in at most 5 and also in at most 10 approaches, $\pi_N(\text{on}, 5)$ and $\pi_N(\text{on}, 10)$ respectively, namely through $(0; 1 - \alpha^{\frac{1}{\pi}})$ as confidence interval and through Bayesian updating.

Firstly, we see from Table 3 that there were altogether 191 approaches so that the 99% confidence interval for $\pi = P(\text{success in a single approach})$ is $(0; 1 - \alpha^{\frac{1}{\pi}}) = (0.0238)$. From this confidence interval and using the relation $p_N(\text{on}, n) = F_N(n) = 1 - (1 - p_N(\text{on}, 1))^n$, we derive the 99% confidence intervals for $\pi_N(\text{on}, 5)$ and $\pi_N(\text{on}, 10)$ respectively as $(0; 0.113)$ and $(0; 0.214)$.

Secondly, we found Bayesian point and interval estimates of $\pi_N(\text{on}, 5)$ and $\pi_N(\text{on}, 10)$, using the same line of reasoning as above. Reverting to Assumption A, we may look at Experiment 1 as a Bernoulli ($\pi_N(\text{on}, 1)$) process consisting of 528 independent trials. We then assume a beta prior for $\pi_N(\text{on}, 1)$ with parameters $\alpha$ and $\beta$, estimated from $p_N(\text{on}, 1) = \frac{8}{528} = 0.0152 = \frac{\alpha}{\alpha + \beta}$ and (standard error)$^2 = 0.0152(1 - 0.0152)/528 = \frac{\alpha \beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$ to obtain estimates $\hat{\alpha} = 7.9849$ and $\hat{\beta} = 519.015$. Combining the beta prior with the likelihood function of a Bernoulli process with $n = 191$ trials and 0 successes, we found a beta posterior with parameters $a = \hat{\alpha} + \sum x_i = 7.9849$ and $b = \hat{\beta} + n - \sum x_i = 710.015$. These estimates lead us to a Bayes point estimate and a 99% lower Bayes interval for $\pi_N(\text{on}, 1)$: $p_N(\text{on}, 1) = 0.011121$ and $(0, 0.0219034)$. From these
point and interval estimates, again using the relation \( p_N(on, n) = F_N(n) = 1 - (1 - p_N(on, 1))^n \), we found \( p_N(on, 5) = 0.054 \) and a 99% lower Bayes confidence interval \((0; 0.105)\). Similarly, we found \( p_N(on, 10) = 0.106 \) with 99% lower Bayes confidence interval \((0; 0.199)\).

We summarized all the calculated probabilities and standard errors/confidence intervals after the second experiment was conducted in Table 4.

**Table 4.** Estimated probabilities of successful feeding and standard errors (power-off case) or 99% confidence intervals (power-on case) after Experiment 2.

<table>
<thead>
<tr>
<th>Power-off</th>
<th>Power-on</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Naive</td>
</tr>
<tr>
<td>(t)</td>
<td>(p_T(\text{off}, t)): Point estimate and standard error</td>
</tr>
<tr>
<td>5</td>
<td>0.708</td>
</tr>
<tr>
<td>10</td>
<td>0.958</td>
</tr>
<tr>
<td>(n)</td>
<td>(p_N(\text{off}, n)): Point estimate and standard error</td>
</tr>
<tr>
<td>5</td>
<td>0.833</td>
</tr>
<tr>
<td>10</td>
<td>0.958</td>
</tr>
</tbody>
</table>

### 4.3 Interpretation of probabilities

**Comparison of two experiments**

For the probability of a successful feeding event in at most 5 and at most 10 minutes, we found a fair agreement between the estimated values calculated after the two experiments respectively and for both modes. Although there were no successful feeding events in power-on mode for all 24 trials in the second experiment, it appears as if the number of trials in this experiment were insufficient to really make a difference through Bayesian updating.
A similar comparison of the probabilities in at most 5 and 10 approaches cannot be done for power-off mode. For power-on mode, it appears as if the probability estimates \( p_N(\text{on}, 5) \) and \( p_N(\text{on}, 10) \) are somewhat smaller after Experiment 2.

**Comparison of two power modes**

In summary, we may fairly safely conclude about the location of the probabilities that

\[
\begin{align*}
\pi_T(\text{off}, 5) &= 0.7 \quad \text{and} \quad \pi_T(\text{on}, 5) = 0.08 \\
\pi_T(\text{off}, 10) &= 0.9 \quad \text{and} \quad \pi_T(\text{on}, 10) = 0.16
\end{align*}
\]

These values indicate a dramatic difference between the chances of a shark attack in the two power modes (a reduction of more than 80% from power-off to power-on mode in both cases). Although the probability of an attack in either mode is increased when going from 5 to 10 minutes, the odds of an attack in off mode relative to that in on mode is higher for 10 minutes than for 5 minutes. This comparison can be formalised by using the **odds ratio**. For a five minutes time period, this odds ratio is \( \frac{0.7/0.3}{0.08/0.92} = 26.8 \), and for a ten minutes time period, the odds ratio is \( \frac{0.9/0.1}{0.16/0.84} = 47.3 \). Similarly, for a comparison of numbers of approaches to feeding, the odds ratio’s (power-off to power-on) are \( \frac{0.8/0.2}{0.05/0.95} = 76 \) and \( \frac{0.96/0.04}{0.11/0.89} = 194.2 \) for 5 and 10 approaches respectively.

### 5. Discussion

Statistical analysis is needed to give scientific support in an ongoing research program on the prevention of shark attacks at the coastal shores of South Africa and other countries. To this end, properly planned experiments together with data capturing of high quality are extremely important factors. We feel that the data used in this research fulfill to a great extent these requirements and enabled
us to apply statistical techniques in estimating shark attack probabilities under varying experimental conditions.

It is important that the uniqueness of the experimental situation surrounding these tests be highlighted. White sharks visiting Dyer Island do so to feed on the Cape fur seals moving between open water and the colony on the adjacent Geyser Rock. As such, these sharks are probably in a heightened predatory mode and therefore more likely to approach and attack potential food than when they are merely travelling or in another behavioural state. Additionally, the sharks were attracted to the baits using a chum slick consisting of ground fish mixed with sardine *Sardinops sagax* oil, thereby increasing the likelihood of attack on the bait. It is unlikely that divers would present such a degree of “attractability” to sharks, but it was believed that testing of the SharkPOD™ device under such heightened predatory conditions would simulate a “worst-case” scenario for divers. Divers using the SharkPOD™ under conditions where sharks are present but not actively feeding may have the assurance that the chances of being attacked could even be less than the reported figures given in this research.

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References


